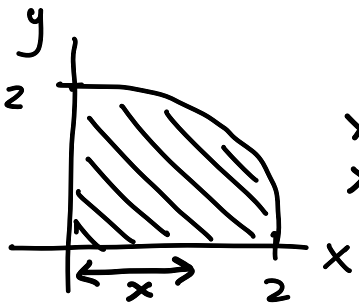
Closing Thu: 15.2
Closing Tue: 10.3, 15.3

Entry Task: (15.2 HW Problem 12)
Find the volume of the solid in the first octant bounded by

$$x^2 + y^2 = 4, y = 3z, \text{and } z = 0.$$

$z = \frac{1}{3}y \quad z = 0 \quad \iint_R \frac{1}{3}y \, dA$



$x=0 \quad y=0$
$x^2+y^2=4$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{1}{3}y \, dy \, dx$$

$$= \int_0^2 \frac{1}{6}(4-x^2) \, dx$$

$$= \boxed{8/9}$$

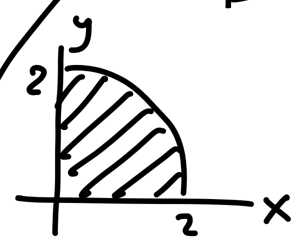
visual: https://www.math3d.org/cJFoVYQf

What if we changed it to the solid in the first octant bounded between

$$x^2 + y^2 = 4, y = 3z, \text{and } z = 1.$$

$z = \frac{1}{3}y \quad z = 1 \leftarrow$ have to do for each z

$\iint_R \frac{1}{3}y \, dA \qquad \iint_R 1 \, dA$



$$\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{1}{3}y \, dy \, dx = \frac{8}{9}$$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} 1 \, dy \, dx = \pi$$

Subtract smaller value from larger value

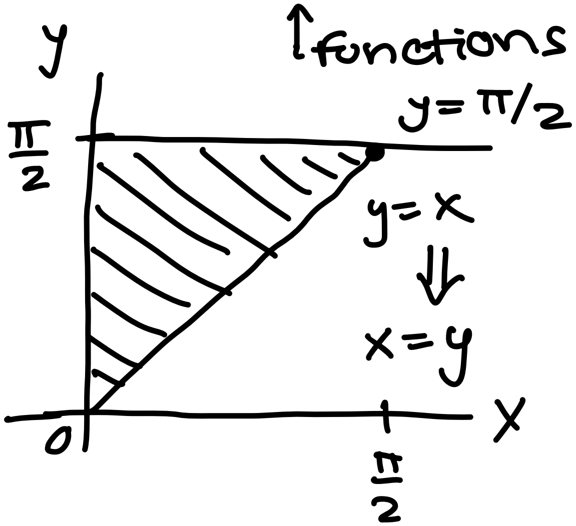
$$\boxed{\pi - \frac{8}{9}}$$

Visual: https://www.math3d.org/bkMaaYxF

Draw the region and switch the order of integration for…

1. $$\int_0^{\pi/2}\int_x^{\pi/2} \frac{\sin(y)}{y}\, dy\, dx$$

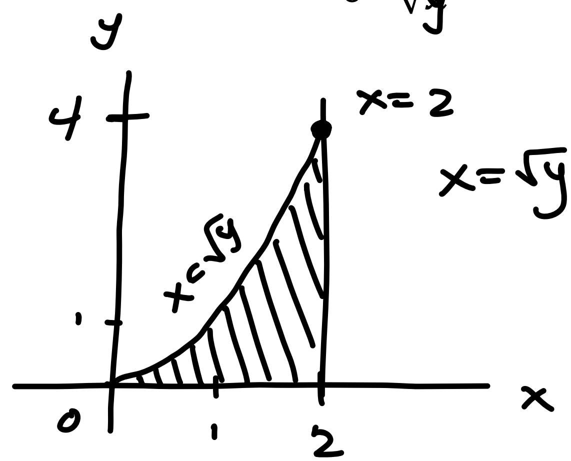
↑functions



$$\int_0^{\pi/2}\left(\int_0^{y} \frac{\sin(y)}{y}\, dx\right) dy$$

easy integration !

2. $$\int_0^{4}\int_{\sqrt{y}}^{2} \sin(x^3)\, dx\, dy$$

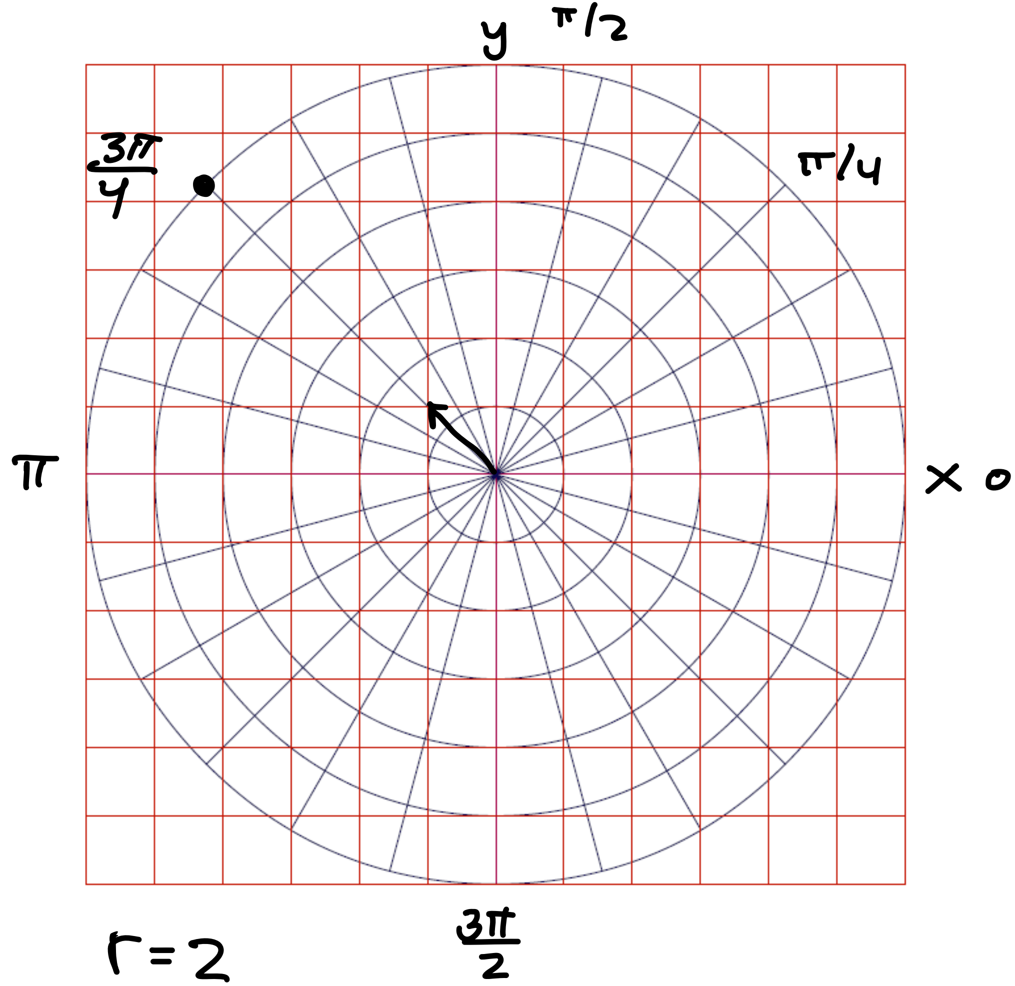


$x=\sqrt{y} \Rightarrow y=x^2$

$$\int_0^2\int_0^{x^2} \sin(x^3)\, dy\,\, dx$$

10.3 Polar Coordinates

Polar
Given (r, θ) 1. Stand at origin facing the positive *x*-axis.
2. Rotate by θ. pos. = ccw, neg. = clockwise
3. Walk *r*-units in direction you are facing. pos. = forward neg. = backward



$r = 2$

$\theta = \frac{3\pi}{4}$

Plot this point (r, θ) = (2, 3π/4)
Give 3 other polar ways to get to this same points

$(-2, -\pi/4)$

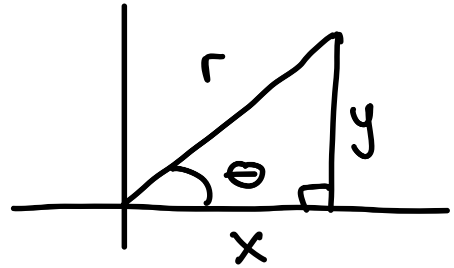
$(2, 3\pi/4 + 2000\pi)$

$= (2, 3\pi/4)$

From trig we already know:

$$x = r\cos(\theta), \qquad y = r\sin(\theta)$$

$$\tan(\theta) = \frac{y}{x}, \qquad x^2 + y^2 = r^2$$



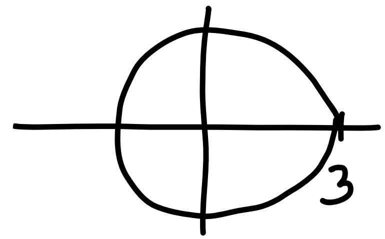
converting r, θ to x, y

Four Quick Exercises:

1. Describe all pts where *r* = 3.

   θ = Anything

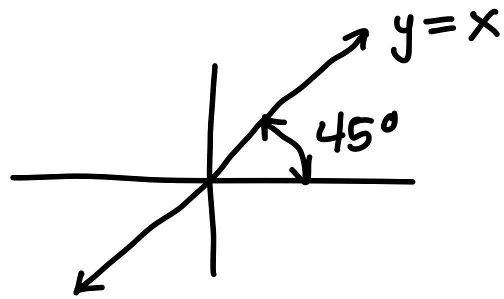
   $\boxed{\text{circle w/ radius 3}}$



2. Describe all pts where θ = π/4.

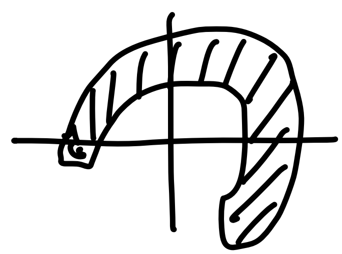
   r = Anything

   $\boxed{\text{a line} \Rightarrow y = x}$



3. Describe all pts where

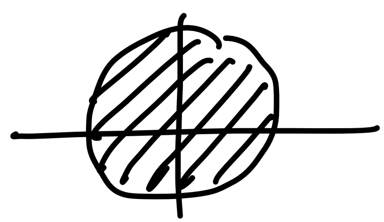
   $$-\frac{\pi}{4} \le \theta \le \pi \text{ and } 1 \le r \le 3$$



numbers insted of functions!

4. Describe all pts where

   $$0 \le \theta \le 2\pi \text{ and } 0 \le r \le 2$$

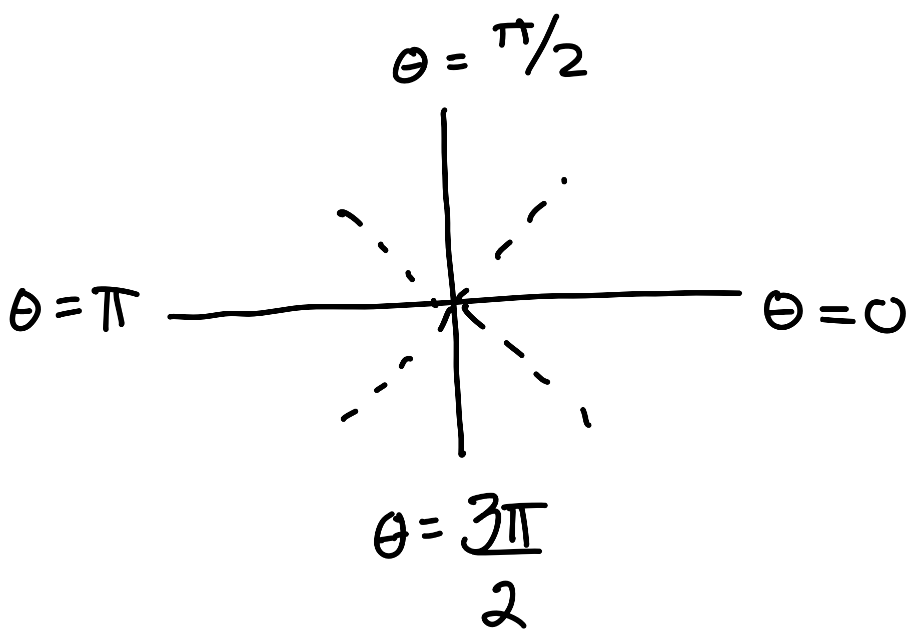


$$\int_0^{2\pi}\int_0^{2} \_\_\_ \, dA$$

Plotting Polar Curves

Option 1: Try to convert to x and y.
Then hope you recognize the curve.

Option 2: **Plot points!**
Start with 0, π/2, π, 3π/2 (intercepts).
For more detail do multiples of π/6
and π/4.



Example: Similar to 10.3 HW Prob 7

Convert to Cartesian: double angle

$$r^2 \sin(2\theta) = 1$$

$x = r\cos\theta \quad y = r\sin\theta$

$x^2 + y^2 = r^2 \quad \tan\theta = y/x$

$2r^2\cos\theta\sin\theta = 1$

$2r\cos\theta\, r\sin\theta = 1$

$\boxed{2xy = 1}$

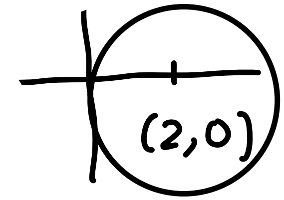
Example: Similar to things in 15.3

Convert to Polar:

$$(x-2)^2 + y^2 = 4$$

$x^2 - 4x + 4 + y^2 = 4$

$x^2 + y^2 - 4x = 0$



$r^2 - 4x = 0$

$\boxed{r^2 - 4r\cos\theta = 0}$

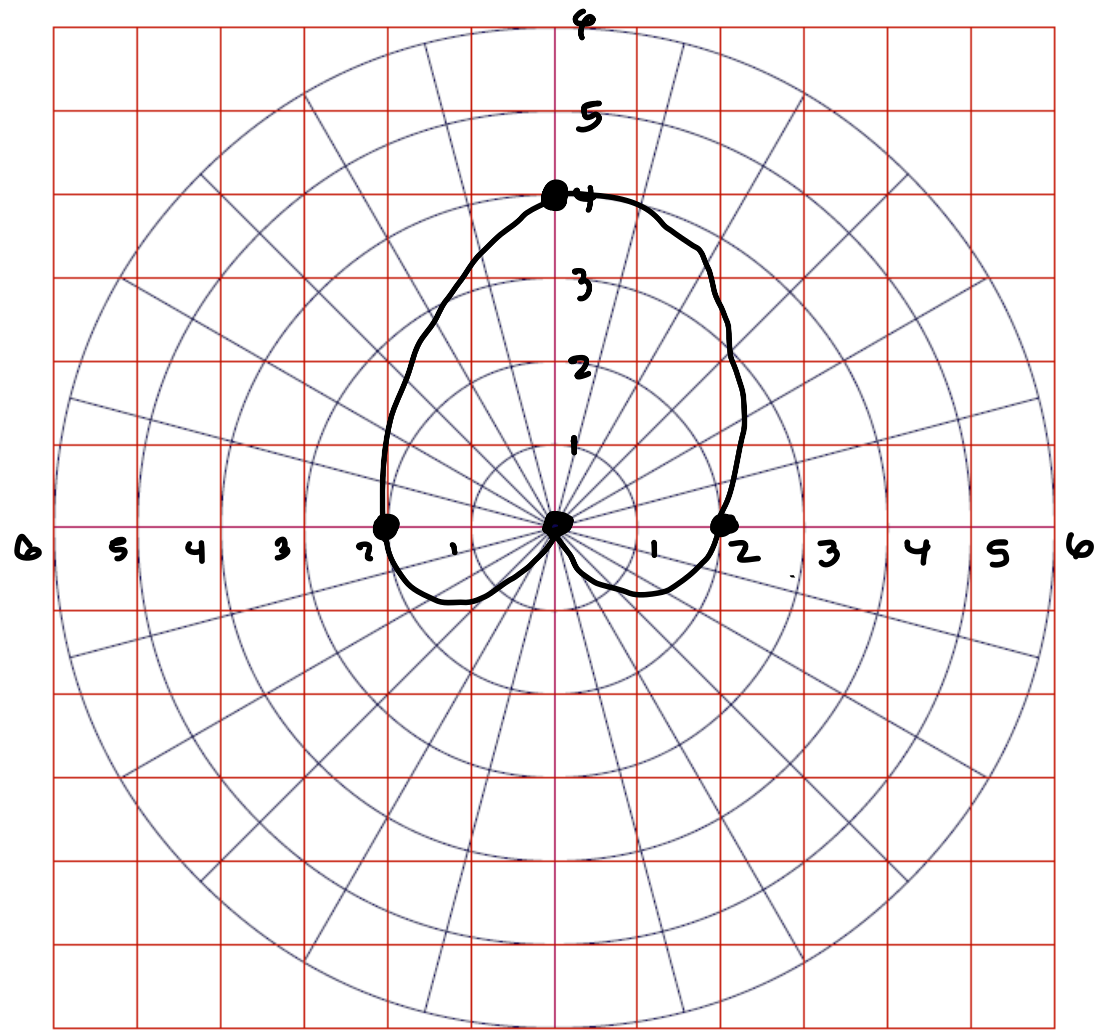
$\boxed{r = 4\cos\theta}$

Example: Like HW 10.3 Prob 7

Graph $r = 2(1 + \sin(\theta))$

θ	0	π/2	π	3π/2	2π
r	2	4	2	0	

θ	π/6	π/4	π/3	2π/3	3π/4	5π/6
r						



Visual: https://www.desmos.com/calculator/87hznx7um8

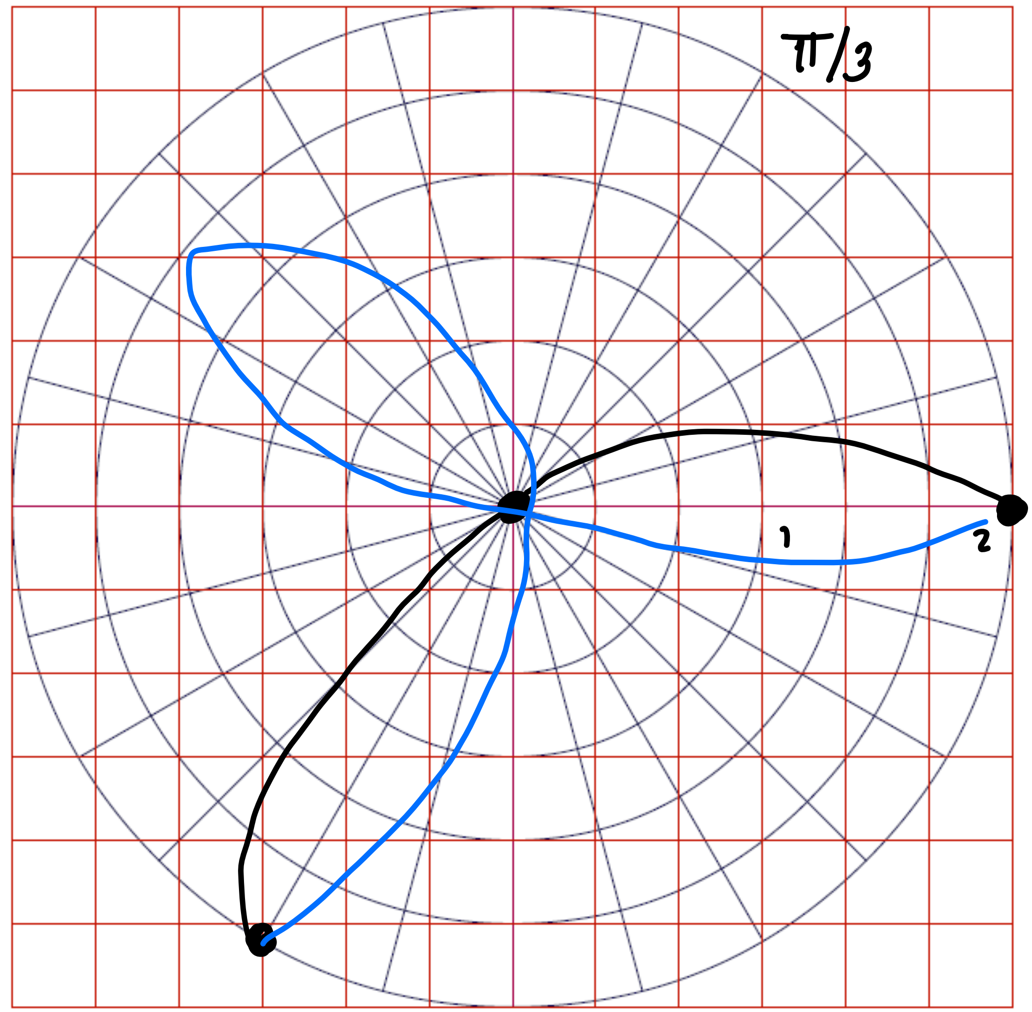
Example: Like HW 15.3 Prob. 5

Graph "one loop" of the "rose"

$$r = 2\cos(3\theta)$$

θ	0	π/2	π	3π/2	2π
r	2	0	-2	0	2

θ	π/6	π/4	π/3	2π/3	3π/4	5π/6
r	0		-2			



Visual: https://www.desmos.com/calculator/bnklo2q3jh

Example: Like 15.3 HW Prob 7

Graph the region inside the cardiod

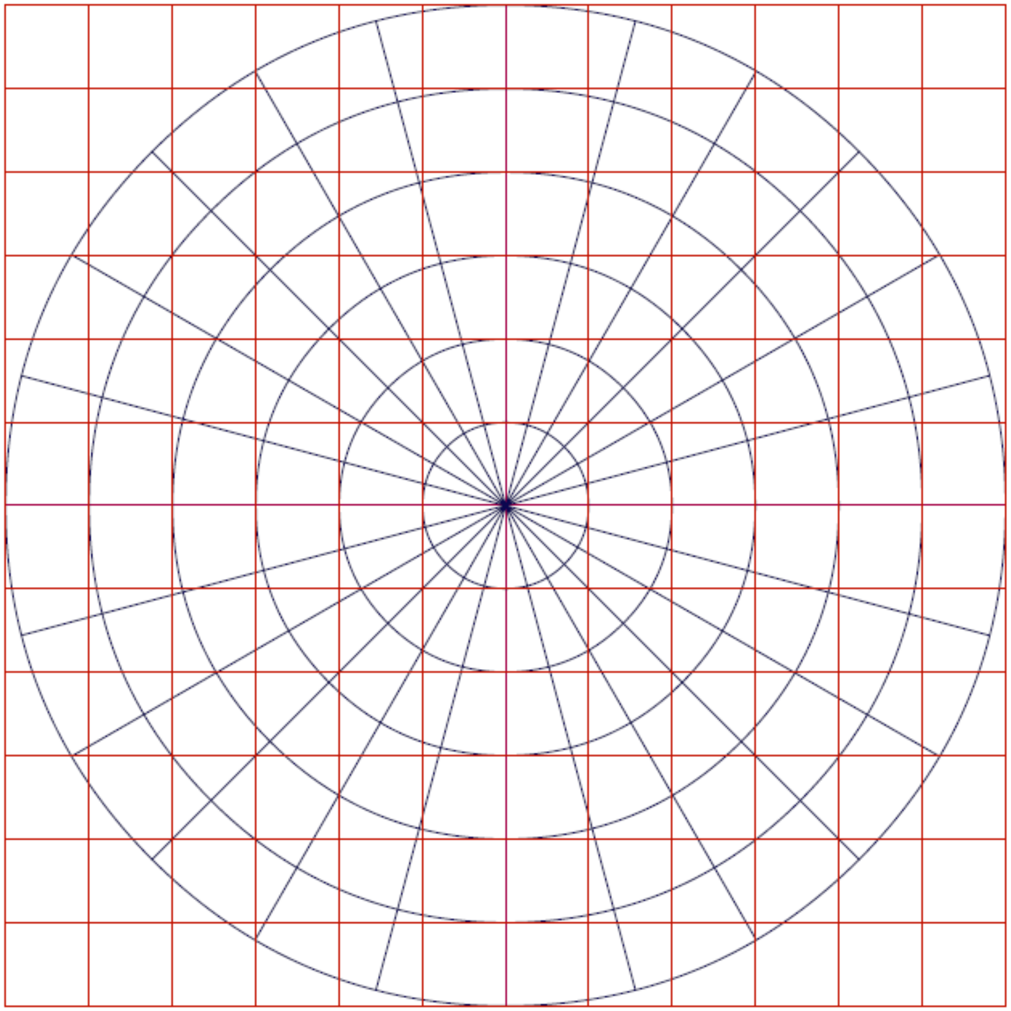
$$r = 1 + \cos(\theta)$$

and outside the circle

$$r = 3\cos(\theta)$$

θ	0	π/2	π	3π/2	2π
r					

θ	π/6	π/4	π/3	2π/3	3π/4	5π/6
r						



Visual: https://www.desmos.com/calculator/wc62p5zmy9